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Average Λ-Nucleon Interaction in the Hypertriton*

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Upper bounds on the strength of the average Λ-nucleon interaction required to account for the observed binding of the hypertriton have been established in a variation calculation with a ten-parameter trial function. Central two-body potentials having hard-core radii 0.2, 0.4, and 0.6 F and ranges suggested by consideration of the two-pion-exchange mechanism were used. These calculations led to improvements of 3–6% (depending upon the hard-core radius) in the required strengths over the results of previous calculations with a four-parameter function. The results cannot be used to rule out the existence of a bound hyperdeuteron of spin 0 for a hard-core radius as large as 0.6 F. A partial variation calculation is described, in which only two of the ten parameters of the trial function are varied. When the remaining parameters are fixed by consideration of the two-body subsystems of which the hypertriton is composed, variation of two appropriate parameters leads to required strengths within 3% of the values obtained by varying all ten parameters.

I. INTRODUCTION

In a recent paper by Downs, Smith, and Truong (referred to hereafter as DST),[1] the results of an analysis of the hypertriton in terms of hard-core potentials were reported. In that study upper bounds on the strength of the average Λ-nucleon interaction required to give the value $B_\Lambda = 0.2$ MeV for the binding energy of the Λ-particle in the hypertriton were established in a variation calculation with a four-parameter trial function. Central two-body potentials having hard-core radii 0.2, 0.4, and 0.6 F were used; these were assumed to include the effect of a possible tensor component. It was found that the strength (measured by the well-depth parameter or the zero-energy scattering length) of the required average Λ-nucleon potential increases as the hard-core radius is increased. For a hard-core radius 0.6 F, the strength of the required Λ-nucleon interaction was found to be consistent with the existence of a bound hyperdeuteron of spin 0, but not of spin 1. If the Λ-nucleon system does form a bound state, it would be expected to have spin 0 because the singlet Λ-nucleon interaction is now believed to be more attractive than the triplet.[2]

The four-parameter trial function of DST is a relatively simple one, consistent with the requirements of hard-core potentials, which is capable of providing for an asymmetric structure of the hypertriton.[3] In previous analyses of the hypertriton in terms of potentials without hard cores, the use of a five-parameter trial function led to reductions of about 10–20% in the upper bounds on the strength of the required average Λ-nucleon interaction obtained with a simple two-parameter trial function.[4,5] This suggests that the use of a trial function of greater flexibility than that used by DST might lead to a significant reduction in the required strength of the average Λ-nucleon interaction in the presence of hard cores. Since the hypertriton is a major source of information on the Λ-nucleon interaction, it is important to know whether such a refinement in the analysis of DST is possible. In particular, if reductions of about 10% could be obtained in the strengths of the required Λ-nucleon interactions, then none of the hard-core potentials considered by DST would imply a bound state of the hyperdeuteron.

It is the purpose of this paper to report the results of a variation calculation of the hypertriton in terms of a ten-parameter trial function. The hard-core potentials considered by DST were used; and this paper is essentially an extension of that work. The results of the ten-parameter variation calculation are given in Sec. II. Improvement of about 3–6% (depending upon the hard-core radius) over the results of DST were obtained; and the improvement for the hard-core radius 0.6 F is not enough to rule out the existence of a bound hyperdeuteron. In Sec. III a partial variation calcu-

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[1] B. W. Downs, D. R. Smith, and T. N. Truong, Phys. Rev. **129**, 2730 (1963).

[2] For a discussion of the evidence supporting this assignment see R. H. Dalitz, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company Ltd., London, 1961), p. 103; and R. H. Dalitz, "The Strong and Weak Interactions of Bound Λ-Particles", paper presented at the (CERN) International Conference on Hyperfragments, St. Cergue, Switzerland, March, 1963 and subsequently issued as Report EFINS-63-29 by the Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois.

[3] See Ref. 4 for a discussion of attributes required of a hypertriton trial function in order that it lead to a reliable estimate of the strength of the average Λ-nucleon interaction.

[4] R. H. Dalitz and B. W. Downs, Phys. Rev. **110**, 958 (1958).

[5] B. W. Downs and R. H. Dalitz, Phys. Rev. **114**, 593 (1959).

TABLE I. Parameters for the potentials (2).

Potential combination	D (F)	V_0 (MeV)	η (F^{-1})	λ (F^{-1})
1	0.2	286.2	1.895	3.219
2	0.4	475.0	2.521	5.059
3	0.6	947.0	3.676	11.804
4	0.4	475.0	2.521	2.361

lation is described. It is shown that relatively good results can be obtained by varying only one parameter in the four-parameter function of DST and only two parameters in the ten-parameter function of the present work if the remaining parameters are properly selected by consideration of the two-body subsystems of which the hypertriton is composed. A discussion of the results of this study, including an indication of the dependence of the required average Λ-nucleon interaction on the binding energy B_Λ, is given in Sec. IV.

II. THE TEN-PARAMETER VARIATION CALCULATION

In order to improve upon the results of DST, we have made a variation calculation with a ten-parameter trial function of the form

$$\psi=f(r_1)f(r_2)g(r_3) \tag{1a}$$

with

$$\begin{aligned} f(r)&=0, & r<D \\ &=[e^{-\alpha(r-D)}-e^{-\beta(r-D)}] \\ &\quad+x[e^{-\mu(r-D)}-e^{-\nu(r-D)}], & r>D \end{aligned} \tag{1b}$$

and

$$\begin{aligned} g(r)&=0, & r<D \\ &=[e^{-\gamma(r-D)}-e^{-\delta(r-D)}] \\ &\quad+y[e^{-\epsilon(r-D)}-e^{-\xi(r-D)}], & r>D, \end{aligned} \tag{1c}$$

where r_1 and r_2 are the Λ-nucleon separations, r_3 is the nucleon-nucleon separation, and D is the hard-core radius. The four-parameter trial function used by DST can be obtained from (1) by setting $x=y=0$.

We have assumed nucleon-nucleon potentials of the form

$$\begin{aligned} V(r)&=\infty, & r<D \\ &=-V_0e^{-\eta(r-D)}, & r>D \end{aligned} \tag{2a}$$

and Λ-nucleon potentials of the form

$$\begin{aligned} U(r)&=\infty, & r<D \\ &=-U_0e^{-\lambda(r-D)}, & r>D \end{aligned} \tag{2b}$$

TABLE II. Results of the five parameter deuteron calculation.

D (F)	V_0 (MeV)	η (F^{-1})	γ (F^{-1})	δ (F^{-1})	ϵ (F^{-1})	ξ (F^{-1})	y	B_d (MeV)
0.2	286.2	1.895	0.971	5.46	0.398	3.80	0.614	2.145
0.4	475.0	2.521	1.193	3.80	0.394	3.90	0.477	2.237
0.6	947.0	3.676	1.606	3.81	0.445	4.28	0.524	2.190

with the same hard-core radius and exponential attractive wells. The parameters V_0 and η of (2a) and the range parameters λ of (2b), which we have used, are given in Table I. The potentials (2) with these parameters were also used by DST, who discussed the choice of range parameters. The range parameter λ is related to the intrinsic range b^0 of the attractive well (that is, the intrinsic range the attractive well would have if it were translated to the origin) by $\lambda=3.5412/b^0$. The intrinsic ranges b^0 of the Λ-nucleon potentials in the first three rows of Table I are related by

$$b^0=1.5\text{ F}-2D;$$

and the intrinsic range of the Λ-nucleon potential in the fourth row is $b^0=1.5$ F, the largest possible consistent with an assumed dominant two-pion-exchange mechanism.

Upper bounds on the depth U_0 of the average Λ-nucleon potential (2b) were established for the potential combinations listed in Table I by the variational procedure described by DST. The formulation of the variational problem given by DST can easily be extended for the case of the ten-parameter trial function (1).[6]

TABLE III. Results of the five-parameter "hyperdeuteron" calculation.

D (F)	λ (F^{-1})	α (F^{-1})	β (F^{-1})	μ (F^{-1})	ν (F^{-1})	x	U_0 (MeV)
0.2	3.219	1.127	6.05	0.215	7.46	0.362	644.2
0.4	5.059	1.685	5.08	0.268	6.73	0.375	1582
0.6	11.804	1.906	9.86	0.267	10.01	0.491	8514

Initial values for the variation parameters $(\gamma,\delta,\epsilon,\xi,y)$ were obtained from variation calculations of the binding energy B_d of the deuteron using the two-body trial function (1c) and the nucleon-nucleon potentials of Table I. The optimum parameters and the corresponding values of B_d, obtained in these calculations, are given in Table II. The value of B_d given in the second row of Table II is slightly greater than the empirical value $B_d=2.225$ MeV. The nucleon-nucleon potentials used in these calculations are those obtained by Ohmura *et al.*[7] for their study of the hypertriton. Since the variation method leads to an upper bound on the binding energy for the potential considered, the excessive binding energy obtained here shows that the corresponding potential is slightly too attractive. Indeed, the values of B_d given in Table II indicate that the trial function (1c) provides a very good representation of the binding

[6] D. R. Smith, thesis, University of Colorado, 1963.

[7] T. Ohmura (Kikuta), M. Morita, and M. Yamada, Prog. Theoret. Phys. (Kyoto) **15**, 222 (1956). The nucleon-nucleon potentials used in this reference were obtained with the help of the interpolation formulas of J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

TABLE IV. Results of the ten-parameter hypertriton calculation.

Potential combination	α (F^{-1})	β (F^{-1})	μ (F^{-1})	ν (F^{-1})	x	γ (F^{-1})	δ (F^{-1})	ϵ (F^{-1})	ξ (F^{-1})	y	U_0 (MeV)
1	0.842	4.30	0.126	8.95	0.769	0.910	5.75	0.351	2.70	0.434	401.7
2	1.17	5.08	0.141	6.07	0.907	1.24	3.67	0.386	3.93	0.530	1144
3	1.79	9.22	0.171	9.87	1.10	1.84	3.44	0.421	3.98	0.398	7123

energy of the deuteron for the nucleon-nucleon potentials of Table I.

Initial values for the variation parameters (α,β,μ,ν,x) were determined by minimizing the depth U_0 of the Λ-nucleon potentials (2b) required to produce a fictitious hyperdeuteron having zero binding energy with the two-body trial function (1b). The optimum parameters and the corresponding values of U_0 obtained in these calculations are given in Table III for the first three sets of Λ-nucleon potential parameters in Table I. These values of U_0 are about 4–8% smaller than the corresponding values obtained by DST in a two-parameter "hyperdeuteron" calculation; they are included primarily for this comparison.

Starting with the values of the variation parameters given in Tables II and III, the depth U_0 of the average Λ-nucleon potential (2b) required to give a binding energy $B_\Lambda=0.2$ MeV for the Λ-particle in the hypertriton was minimized with respect to the ten parameters in (1) by the iteration technique described by DST. The results of these calculations are summarized in Table IV for the potential combinations of Table I. With the large number of variation parameters used in these calculations, a change in one parameter can be partially compensated for by a change in one or more of the other parameters with relatively little effect on the value of U_0 in the vicinity of its minimum. Consequently, the values of the parameters given in Table IV may not be the optimum sets, even though the values of U_0 given in Table IV are expected to be quite close to the minimum values for the trial function (1). A similar calculation for the potential combination 4 led to the depth $U_0=226.1$ MeV for the long-ranged Λ-nucleon potential.

A comparison of the results of these calculations with the results of the four-parameter calculations of DST is given in Table V. The well depths U_0, the well-depth parameters s, the zero-energy scattering lengths a, and the effective ranges r_0 of the average Λ-nucleon potentials are given in both cases. The well depths U_0 (and the well-depth parameters s) obtained with the ten-parameter function (1) are about 3–6% less than those obtained by DST with a four-parameter function, the greatest improvement being obtained for the smallest hard-core radius. These improvements are appreciably less than those obtained by Downs and Dalitz with a five-parameter trial function[5] over the results of their two-parameter calculations[4] for potentials without hard cores.

The improvement attained here is not sufficient to rule out the existence of a bound hyperdeuteron for a hard-core radius of 0.6 F. If the two-body central potentials considered here are appropriate for a discussion of the two-body Λ-nucleon system, then the well-depth parameter $s=0.925$ obtained here would imply a bound hyperdeuteron of spin 0 unless the ratio s_t/s_s of the well-depth parameters for the triplet and singlet potentials is at least 0.7,[1,8] which is considerably larger than existing estimates (0.45–0.55).[4,9] If attractive three-body potentials make a significant contribution to the binding of the hypertriton, the two-body potentials would be weaker than our calculations indicate.[10,11] In this case, the hyperdeuteron might not be expected to be bound even for the largest hard-core radius considered here. The potentials used here have been assumed to contain the effect of a possible tensor component. If a tensor force is present and makes an attractive contribution to the average Λ-nucleon interaction in the hypertriton, the actual central potential

TABLE V. Comparison of the results of the four-parameter and ten-parameter calculations.

Potential combination	Four-parameter (DST) U_0 (MeV)	s	a (F)	r_0 (F)	Ten-parameter U_0 (MeV)	s	a (F)	r_0 (F)
1	426.0	0.744	−2.20	2.13	401.7	0.701	−1.75	2.31
2	1202	0.851	−2.56	2.01	1144	0.810	−1.82	2.24
3	7352	0.955	−4.09	1.80	7123	0.925	−2.11	2.10
4	234.5	0.762	−3.20	3.32	226.1	0.734	−2.71	3.56

[8] If the singlet Λ-nucleon interaction is more attractive than the triplet, as is believed to be the case (see footnote 2), the singlet well-depth parameter is related to the average well-depth parameter s obtained here by $s_s=s[3/4+s_t/4s_s]^{-1}$.

[9] K. Dietrich, R. Folk, and H. J. Mang, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company Ltd., London, 1961), p. 165.

[10] A. R. Bodmer and S. Sampanthar, Nucl. Phys. **31**, 251 (1962).

[11] J. D. Chalk, III, and B. W. Downs, Phys. Rev. **132**, 2727 (1963), and other references cited there.

TABLE VI. Results of partial variation calculations in which only the parameters α and x were varied, the values of the other parameters in (1) being those given in Tables II and III.

Potential combination	α (F^{-1})	x	U_0 (MeV)
1	1.02	1.93	412.1
2	1.97	1.37	1161
3	2.44	1.71	7180

which would determine the existence of a hyperdeuteron of spin 0 would be correspondingly weaker; and a bound two-body system would still be in question.

The low-energy scattering parameters, obtained with the ten-parameter trial function and listed in the first three rows of Table V, are not very sensitive to the value of the hard-core radius. Moreover, for potentials which have well-depth parameters as close to unity as these do, small changes in the depths U_0 lead to relatively much larger changes in a and r_0.[12] For a potential without a hard core, having an intrinsic range 1.5 F [see Eq. (3)], Downs and Dalitz[4] obtained the values $a\approx -1.5$ F and $r_0\approx 2.8$ F for the scattering length and effective range of the average Λ-nucleon interaction in the hypertriton. If Eq. (3) is a reasonable way of determining the range parameter of the attractive well for a simple hard-core potential, then scattering parameters in the ranges $a\approx -(1.5\text{–}2.1)$ F and $r_0\approx(2.1\text{–}2.8)$ F may provide a good approximate characterization of the average Λ-nucleon interaction for any hard-core radius in the range $D=0-0.6$ F. The scattering parameters given in the fourth row of Table V fall well outside these limits. If the longer range, to which these values correspond, turns out to be correct, a more careful characterization than that suggested above will be required for each possible hard-core radius.

III. A PARTIAL VARIATION CALCULATION

The four-parameter trial function of DST is that given in Eqs. (1) with $x=y=0$. It was noted by DST that the optimum values of the parameters γ and δ obtained in their hypertriton calculation are quite close to the values of these parameters which lead to the maximum value of B_d in a variation calculation of the binding energy of the deuteron with the nucleon-nucleon part of their trial function. Moreover, the optimum value of the parameter β is quite close to the value obtained in a two-body Λ-nucleon calculation analogous to that summarized here in Table III. This suggested that the three parameters γ, δ, and β could be fixed by two-body calculations, and only the parameter α varied in the three-body calculation. When this was done, it led to values of U_0 within one percent of the values obtained (see Table V) when all four parameters were varied.[6]

The success of the partial variation calculation mentioned above suggested that the results of our ten-parameter variation calculation could be approximated by a calculation in which only a small number of the parameters of the trial function (1) were varied. A partial variation calculation was therefore made in which the values of all parameters except α and x were fixed at the values given in Tables II and III. Variation of α and x then led to the values given in Table VI for the first three potential combinations in Table I. The value of U_0 in the first row of Table VI is within 2.6% of the corresponding value in Table IV obtained with the variation of all ten parameters; the values of U_0 in the second and third rows of Table VI are appreciably closer to the corresponding values in Table IV. It is not surprising that the value of U_0 for $D=0.2$ F obtained in this partial variation calculation should be relatively worse than the others. It was for this case that the full variation of ten parameters led to the greatest improvement over the four-parameter results of DST, indicating that the flexibility provided by variation of a large number of parameters is required to give good results for the small hard-core radius.

The values of the depths U_0 reported in Table VI are better than those obtained with the variation of all parameters in the four-parameter calculation of DST (see Table V). Since the difference between the results of these two calculations is only about 3%, however, it should be noted that the two-parameter partial variation leading to Table VI required appreciably more effort than the four-parameter variation reported by DST.

IV. CONCLUDING REMARKS

The relatively small improvements (3–6%) in the values of the well depths U_0 obtained with the ten-parameter trial function over the four-parameter results indicate that the four-parameter function of DST can reasonably be used in rough calculations of the binding energy of the hypertriton. Moreover, the results mentioned at the beginning of Sec. III show that the four-parameter results can be well approximated by a variation of only one of the four parameters. The four-parameter function cannot, however, be used to investigate details of the average Λ-nucleon interaction which correspond to changes in U_0 by only a few percent.

The four-parameter function cannot, for example, be used to make an accurate estimate of the dependence of U_0 on the binding energy B_Λ of the Λ-particle in the hypertriton. The value of B_Λ is not well established, the current empirical values being distributed in the range $B_\Lambda=0\text{–}0.4$ MeV.[13] The calculations reported so far in this paper and in the previous one by DST were made for the central value $B_\Lambda=0.2$ MeV. We have also used

[12] Compare the relative changes in the four-parameter and ten-parameter results in Table V. See also Tables VII and VIII of Ref. 1.

[13] A summary of the current binding-energy data was given by R. Levi Setti at the (CERN) International Conference on Hyperfragments, St. Cergue, Switzerland, March 1963.

the four-parameter function of DST to calculate the required values of U_0 for the extreme values $B_\Lambda=0$ and $B_\Lambda=0.4$ MeV. The results of these calculations, which were obtained by varying all four parameters, are given in Table VII along with the values obtained by DST for $B_\Lambda=0.2$ MeV. The differences between the extreme values of U_0 for values of B_Λ in this range are comparable with the differences between the results of the four-parameter and the ten-parameter calculations. The values in Table VII cannot therefore be considered to be quantitatively correct[14]; they are included only as an indication of the magnitude of the variation of U_0 with B_Λ.

We have also calculated U_0 for $B_\Lambda=0$ and 0.4 MeV for the potential combination 2 with $D=0.4$ F by the two-parameter partial variation technique described in connection with Table VI, and obtained the results $U_0(B_\Lambda=0)=1149$ MeV and $U_0(B_\Lambda=0.4\text{ MeV})=1172$ MeV. This variation of U_0 with B_Λ is about the same as that reported in the second row of Table VII, and is comparable with the differences between the values of $U_0(B_\Lambda=0.2\text{ MeV})$ obtained in the full and partial ten-parameter variation calculations (see Tables IV and VI). Therefore, if a quantitatively correct estimate of the energy dependence of U_0 is to be made with the trial function (1), a more complete variation calculation will presumably be required.[15]

TABLE VII. Energy dependence of the well depth U_0 obtained with the four-parameter trial function of DST.

Potential combination	U_0 (MeV) $B_\Lambda=0$	$B_\Lambda=0.2$ MeV	$B_\Lambda=0.4$ MeV
1	419.8	426.0	432.6
2	1192	1202	1212
3	7325	7352	7367
4	228.9	234.5	239.0

Finally, it is noted that Müller has recently completed a calculation of U_0 for an average Λ-nucleon potential essentially the same as that which led to the fourth row of Table V.[16] With an eight-parameter trial function of flexibility comparable to that of (1), he obtained a value of U_0 in substantial agreement with the value 226.1 MeV reported in the fourth row of Table V. The eight-parameter function used by Müller appears to make more economical use of its variation parameters than does (1); and Müller's function probably eliminates some of the parameter redundancy mentioned in connection with Table IV.

[14] When these values of U_0 are used to establish the coefficients m and n in the relation

$$U_0(B_\Lambda)=U_0(0)[1+mB_\Lambda^{\frac{1}{2}}+nB_\Lambda],$$

values $n>m$ are obtained (when B_Λ is expressed in MeV) except for the potential combination 3 with $D=0.6$ F. One would expect that a proper description of the energy dependence for small B_Λ would lead to values $m>n$; see Ref. 5.

[15] See footnote 14. The values of $U_0(B_\Lambda)$ obtained with the two-parameter partial variation calculation for the potential combination 2 also lead to $n>m$.

[16] Dr. D. Müller (private communication). Dr. Müller's eight-parameter function is described in D. Müller, Z. Physik **171**, 371 (1962).